Formeln HF III

Antennas

wave detachment:

$$\underline{U} = \underline{U}_h e^{-j\frac{2\pi}{\lambda}z} \qquad \underline{U}_h = |U_h| e^{j\varphi_h}$$

$$v = Re\{\underline{U} e^{j\omega t}\} = |U_h| \cos(\omega t - \frac{2\pi}{\lambda} z + \varphi_h) = i \cdot Z_L$$

fundamental parameters: radiation characteristics

$$\lim_{r\to\infty} \frac{E}{H} = Z_{F0} = 120\pi\,\Omega \quad \text{... characteristic wave impedance}$$

$$\underline{C}(\vartheta,\varphi) = \frac{\underline{E}(\vartheta,\varphi)}{E_{max}} \rightarrow C(\vartheta,\varphi) = \frac{|E(\vartheta,\varphi)|}{|E_{max}|} = \sqrt{\frac{S(\vartheta,\varphi)}{S_{max}}}$$

$$S_{max} = \frac{1}{2}\frac{|E_{max}|^2}{Z_{F0}} = \frac{1}{2}|I_a|^2 R_a \frac{G_a}{4\pi r^2}$$

$$\rightarrow G_a = \frac{4\pi r^2}{Z_{F0} R_a} \frac{|E_{max}|^2}{|I_a|^2}$$

gain: $G = \frac{S_{max}}{S_I}\Big|_{P_{t0}=const.}$ $\qquad S_I = \frac{P_{t0}}{4\pi r^2}$ $\qquad S_{max}$

directivity: $D = \frac{S_{max}}{S_I}\Big|_{P_t=const.}$ $\qquad G = \eta \cdot D$ $\qquad \eta = \frac{P_t}{P_{t0}}$

$$P_t = \int S\,dA = \int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi} S_{max} \cdot C^2(\vartheta,\varphi)\, r^2 \sin\vartheta\, d\varphi\, d\vartheta$$

$$\rightarrow D = \frac{4\pi}{\Omega_A} \qquad \Omega_A = \iint c^2 \sin\vartheta\, d\varphi\, d\vartheta$$

$$= \varphi_{3dB} \cdot \vartheta_{3dB} \quad \text{(highly directive antennas)}$$

receiving antennas

$$P_{r_{max}} = S \cdot A_e \Leftrightarrow \boxed{A_e} = \frac{P_{r_{max}}}{S} \qquad \boxed{A_0} = \frac{P_{r0_{max}}}{S}$$

$$\boxed{= \frac{\lambda_0^2}{4\pi} \cdot G} \qquad \boxed{A_0 = \frac{\lambda_0^2}{4\pi} D}$$

wireless transmission

free space:

$$S = \frac{P_{t0}}{4\pi r^2} \cdot G_1$$

$$P_{r_{max}} = \frac{P_{t0}}{4\pi r^2} \frac{\lambda_0^2}{4\pi} G_1 G_2 \rightarrow \frac{P_{r_{max}}}{P_{t0}} = G_1 G_2 \left(\frac{\lambda_0}{4\pi r}\right)^2$$

$$a = 10\lg\left(\frac{4\pi r}{\lambda_0}\right)^2 - g_1 - g_2 \quad \text{... isotropic radio link attenuation}$$

over perfectly conducting ground

$$S_{max} = \frac{P_{t0} G_t\, h_t^2 h_r^2 4\pi}{\lambda_0 r^4}$$

$$P_{r_{max}} = \frac{P_{t0} G_t G_r h_t^2 h_r^2}{r^4}$$

in general: $\frac{P_{r_{max}}}{P_{t0}} \sim \frac{1}{r^n}$

$$r_{BK} \approx 8{,}4 \cdot \frac{h_t h_r}{\lambda_0} \quad \text{... breakpoint distance}$$

equivalent field sources

$$\oint \vec{H} d\vec{s} = \sum J_i \qquad \oint \vec{E} d\vec{s} = -j\omega\Phi \quad \underset{J_m}{}$$

$$\boxed{\begin{aligned} \vec{J}_* &= \vec{n} \times (\vec{H}_2 - \vec{H}_1) \\ \vec{M}_* &= -\vec{n} \times (\vec{E}_2 - \vec{E}_1) \end{aligned}}$$

Hertzian Vector

$$\boxed{\vec{\Pi}_M} = \iiint_V M(\vec{r}\,') \frac{e^{-j\beta_0 |\vec{r} - \vec{r}\,'|}}{j\omega\mu_0 4\pi |\vec{r} - \vec{r}\,'|} dV$$

$$\boxed{\approx \frac{e^{-j\beta_0 r}}{j\omega\mu_0 4\pi r} \iiint_V M(\vec{r}\,') e^{j\beta_0 \frac{\vec{r} \cdot \vec{r}\,'}{|\vec{r}|}} dV}$$ far-field (*) approximation

(*)

$$\boxed{\vec{\Pi}} = \iiint_V J(\vec{r}\,') \frac{e^{-j\beta_0 |\vec{r} - \vec{r}\,'|}}{j\omega\varepsilon_0 4\pi |\vec{r} - \vec{r}\,'|} dV$$

$$\boxed{\approx \frac{e^{-j\beta_0 r}}{j\omega\varepsilon_0 4\pi r} \iiint_V J(\vec{r}\,') e^{j\beta_0 \frac{\vec{r} \cdot \vec{r}\,'}{|\vec{r}|}} dV}$$ far-field approximation

Field strengths: electric sources:

$$H_{r_1} = 0$$

$$\underline{H}_{\vartheta_1} = -\omega\varepsilon_0\beta_0 \underline{\Pi}_\varphi \qquad \underline{E}_{\varphi_1} = \beta_0^2 \underline{\Pi}_\varphi$$

$$\underline{H}_{\varphi_1} = \omega\varepsilon_0\beta_0 \underline{\Pi}_\vartheta \qquad \underline{E}_{\vartheta_1} = \beta_0^2 \underline{\Pi}_\vartheta$$

magnetic sources:

$$\underline{E}_{r_2} = 0$$

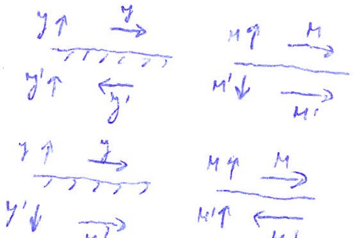
$$\underline{E}_{\vartheta_2} = \omega\mu_0\beta_0 \underline{\Pi}_{m\varphi} \qquad \underline{H}_{\varphi_2} = \beta_0 \Pi_{m\varphi}$$

$$\underline{E}_{\varphi_2} - \omega\mu_0\beta_0 \underline{\Pi}_{m\vartheta} \qquad H_{\vartheta_2} = \beta_0 \Pi_{m\vartheta}$$

image theory

$\kappa \to \infty$:

$\mu \to \infty$:

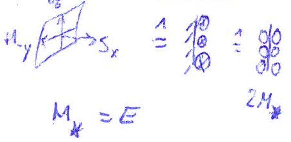


$$\vec{J}_* = \vec{n} \times \vec{H}$$

$$\underbrace{\vec{M}_* = -\vec{n} \times \vec{E}}$$

$$I_m = \int M ds$$

Aperture antennas



$$M_* = E$$

$$\Pi_{my} = \frac{\lambda_0^2}{j\omega\mu_0 2\pi} \frac{e^{-j\beta_0 r}}{r} E_0 \int_{-A/2\lambda_0}^{A/2\lambda_0} G_y\left(\frac{y}{\lambda_0}\right) e^{j2\pi \sin\vartheta \sin\varphi \frac{y}{\lambda_0}} d\left(\frac{y}{\lambda_0}\right) \cdot \int_{-B/2\lambda_0}^{B/2\lambda_0} G_z\left(\frac{z}{\lambda_0}\right) e^{j2\pi\cos\vartheta \frac{z}{\lambda_0}} d\left(\frac{z}{\lambda_0}\right)$$

$$G_y = G_z = 1 \rightarrow \Pi_{my} = \frac{A \cdot B}{j\omega\mu_0 2\pi r} E_0 e^{-j\beta_0 r} \underset{①}{si}\left(\pi \frac{A}{\lambda_0} \sin\varphi \sin\vartheta\right) \underset{②}{si}\left(\pi \frac{B}{\lambda_0} \cos\vartheta\right)$$

$$\rightarrow E_{\vartheta_2} = \beta_0 \frac{AB}{2\pi j r} E_0 e^{-j\beta_0 r} \left[\cos\varphi \, si_① \cdot si_②\right]$$

H-Plane ($\vartheta = 90°$) : $C_H(\varphi) = si\left(\pi \frac{A}{\lambda_0} \sin\varphi\right)$

E-Plane ($\varphi = 0°$) : $C_E(\vartheta) = si\left[\pi \frac{B}{\lambda_0} \sin\left(\frac{\pi}{2} - \vartheta\right)\right]$

Antennas

Aperture antennas

$$\varphi_{3dB} \approx 50{,}4^\circ \cdot \frac{\lambda_0}{A}$$

$$\vartheta_{geo} \approx 50{,}4^\circ \frac{\lambda_0}{B}$$

$$P_t = \frac{1}{2} \frac{|E_0|^2}{Z_{F_0}} AB \rightarrow S_i = \frac{1}{2} \frac{|E_0|^2 AB}{Z_{F_0} \cdot 4\pi r^2}$$

$$S_{max} = \frac{1}{2} \frac{|E_0|^2 (AB)^2}{\lambda_0^2 r^2 Z_{F_0}}$$

$$D = \frac{4\pi}{\lambda_0^2} AB$$

receiving case:

$$A_0 = \underset{\text{aperture efficiency}}{q} \cdot A_{geo} = D \frac{\lambda_0^2}{4\pi} \underset{\substack{\text{uniform distr.}\\(q=1)}}{=} A \cdot B$$

$$A_e = G \cdot \frac{\lambda_0^2}{4\pi} \qquad G = \eta \cdot D$$

Horn antennas

infinite length:

$$E_\vartheta = \frac{2}{\pi} \frac{ABE_0 e^{j\beta_0 r}}{j\lambda_0 r} C(\vartheta, \varphi)$$

$$C(\vartheta,\varphi) = \cos\varphi \frac{\cos(\pi \frac{A}{\lambda_0} \sin\vartheta \sin\varphi)}{1-(2\frac{a}{\lambda_0} \sin\vartheta \sin\varphi)^2} \operatorname{si}\left(\frac{B}{\lambda_0} \cos\vartheta\right)$$

$$\varphi_{3dB} \approx \frac{68{,}9^\circ \cdot \lambda_0}{A} \qquad a_{SL} \approx 23\,dB$$

$$\vartheta_{3dB} \approx \frac{50{,}4^\circ \lambda_0}{B}$$

$$D \approx 0{,}81\, D_{uni} = 0{,}81 \frac{4\pi}{\lambda_0^2} AB \quad (q = 81$$

pyramidical horn

$$L_H^2 = \left(\frac{A}{2}\right)^2 + L^2 \left(\frac{1}{1-\frac{a}{A}}\right)^2 \rightarrow q_H$$

$$L_E^2 = \left(\frac{B}{2}\right)^2 + L^2 \left(\frac{1}{1-\frac{b}{B}}\right)^2 \rightarrow q_E$$

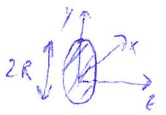
$$P_t = \frac{1}{4} \frac{|E_0|^2}{Z_{F_0}} \sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}\, ab = \frac{1}{4} \frac{|E_0|^2}{Z_{F_0}} \sqrt{1-\left(\frac{\lambda_0}{2A}\right)^2}\, AB$$

$A \gg a$ & $B \gg b$: $Z_{F_H} \rightarrow Z_{F_0}$

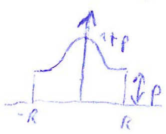
$$|E_0| = \sqrt{\frac{4 P_t Z_{F_0}}{AB}}$$

$$q = \frac{\pi^2}{8} q_E \cdot q_H \approx \frac{q_E \cdot q_H}{0{,}81}$$

Circular antennas



$$E_y(\rho) = E_0 \left(p + \left(1-\left(\frac{\rho}{R}\right)^2\right)^n\right)$$



$$E_y = \left(p + \frac{1}{n+1}\right) \frac{jE_0 \beta_0 R^2}{2} \frac{e^{-j\beta_0 r}}{r} C(\vartheta)$$

$$C(\vartheta) = \frac{1}{p + \frac{1}{n+1}} \left[p \frac{J_1(\beta_0 R \sin\vartheta)}{\beta_0 R \sin\vartheta / 2} + \frac{n!\, J_{n+1}(\beta_0 R \sin\vartheta)}{(\beta_0 R \sin\vartheta / 2)^{n+1}} \right]$$

$$q = \frac{p + \left(\frac{1}{n+1}\right)^2}{p^2 + \frac{2p}{n+1} + \frac{1}{2n+1}}$$

$p=0$, $n=0$:

$$C(\vartheta) = 2 \cdot \frac{J_1(\pi u)}{\pi \cdot u}$$

$$u = \frac{\beta_0}{\pi} R \sin\vartheta = \frac{2R}{\lambda_0} \sin\vartheta$$

$$a_{SL} = 17{,}7\,dB$$

$$\vartheta_{3dB} = 5\cancel{7}^\circ \frac{\lambda_0}{2R}$$

$n=2$:

$$a_{SL} = 30{,}6\,dB$$

$$\vartheta_{3dB} = 84^\circ \frac{\lambda_0}{2R}$$

Reflector antennas

$$q_s = \frac{P_p}{P_t} = \frac{P_p}{\eta \cdot P_0}$$

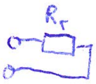
Wire antennas

Dipoles / Monopoles $\boxed{h = \lambda/4}$

$$\vec{\Pi}_z = \frac{e^{-j\beta_0 r}}{j\omega\varepsilon_0 4\pi r} \int_V \vec{J}\, e^{j\beta_0 \frac{\vec{r}\cdot\vec{r}'}{r}}\, dV$$

$$\boxed{l_e = \int_{-h}^{h} \frac{I(z')}{I_A}\, dz'}$$

$\lambda/2$-dipol: $l_e = \frac{\lambda_0}{\pi}$



magnet current source:

$E_\vartheta \to H_\vartheta$

$H_\varphi \to -E_\varphi$

$Z_F \to 1/Z_F$

(magnetic dipole along z-axis)

$$\rightarrow |H_\varphi| = \left|\frac{j2I_m}{2\pi Z_F} \frac{e^{-jkr}}{r}\right| = \frac{I_m}{\pi Z_F r}$$

$$|E_\vartheta| = \left|\frac{jI_m}{\pi} \frac{e^{-jkr}}{r}\right| = \frac{I_m}{\pi r}$$

$$E_\vartheta = j I_A Z_{F0} \frac{e^{-j\beta_0 r}}{2\pi r} \cdot C(\vartheta)$$

$$C(\vartheta) = \left|\frac{\cos\left(\frac{\pi}{2}\cos\vartheta\right)}{\sin\vartheta}\right|$$

$$P_t = \oint \frac{1}{2} \frac{|E_\vartheta|^2}{Z_{F0}} \approx \frac{1}{2} |I_A|^2 R_r \rightarrow 73\,\Omega \text{ (dipole)} \rightarrow 36{,}5\,\Omega \text{ (monopole)}$$

$$\eta \approx 100\%$$

$$D = G = 2{,}15\,dBi$$

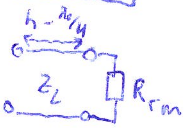
$h = \frac{\lambda}{2}$:

$h = \lambda$

$$I(z') = I_{max} \sin\left[\beta_0 (h - |z'|)\right]$$

$$C(\vartheta) = \left|\frac{\cos(\beta_0 h \cos\vartheta) - \cos(\beta_0 h)}{\sin\vartheta\left[1 - \cos(\beta_0 h)\right]}\right|$$

$\boxed{h > \frac{\lambda_0}{4}}$

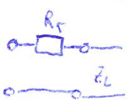


*) R_{rm} ~~from~~ diagram

*) R_{rm} in S.D.
→ um $h - \frac{\lambda_0}{4}$ drehen

$$Z_a = \frac{1 + j \frac{Z_L}{R_{rm}} \tan\left[\beta_0\left(h - \frac{\lambda_0}{4}\right)\right]}{1 + j \frac{R_{rm}}{Z_L} \tan\left[\beta_0\left(h - \frac{\lambda_0}{4}\right)\right]}$$

$\boxed{h < \frac{\lambda_0}{4}}$



*) R_r from diagram

*) $X_a = -Z_L \frac{1}{\tan\beta h}$

$$\boxed{Z_L \approx 120\,\Omega\left[\ln\left(\frac{h}{g}\right) - 0{,}8\right]}$$

folded dipol: $r_e = \sqrt{dg}$ ... diameter of equ. dipol

$$Z_a = 4 Z_d \approx 292\,\Omega$$

HF III Formeln

Antennas

Wire antennas

small dipoles (elementary ~)

$l_e = h$

$$\vec{\Pi} = \frac{I_a \cdot \vec{l}_e}{j\omega\varepsilon_0 4\pi r} e^{-j\beta_0 r} \rightarrow E_{\vartheta,\text{farfield}} = jZ_{F0} \frac{\beta_0 I_a l_e}{4\pi r} \sin\vartheta \, e^{-j\beta_0 r}$$

$$I_a \cdot l_e = j\omega\varepsilon_0 I_m \cdot$$

$$\vec{\Pi}_m = \frac{I_m \cdot \vec{\Delta}}{j\omega\mu_0 4\pi r} e^{-j\beta_0 r} \rightarrow H_{\vartheta,\text{farfield}} = \frac{j\beta_0 I_m \cdot \Delta}{Z_{F0} \cdot 4\pi r} \sin\vartheta \, e^{-j\beta_0 r}$$

$$I_m \cdot \Delta = j\omega\mu_0 I \cdot A$$

$$D_{Hertz} = 1{,}5 \mathrel{\hat{=}} 1{,}8\,dB$$

$$C(\vartheta,\varphi) = \left| \frac{\cos\left(\frac{\pi}{2}\cos\vartheta\right)}{\sin\vartheta} \right|$$

$$R_r = 80\pi^2 \left(\frac{l_e}{\lambda_0}\right)^2$$

Antenna arrays

*) $I_1 = I_2 \cdot e^{j\psi}$

$$|\Delta\vec{r}| = \frac{\vec{r}_1 \cdot \vec{r}_2'}{|\vec{r}|}$$

identical antennas

$$E_{\vartheta_1} = E_{\vartheta_2} \cdot e^{j\delta} \qquad \delta = \psi - \beta_0 \cdot |\Delta\vec{r}| = \psi - \beta_0 d \sin\vartheta \cos\varphi$$

$$C_g = \left|\cos\left(\frac{\delta}{2}\right)\right|$$

$$|E_{\vartheta_a}| = \cdot \, |E_{\vartheta_{2\,max}}| \cdot \underbrace{C_S(\vartheta,\varphi) \cdot C_g(\vartheta,\varphi)}_{C_a(\vartheta,\varphi)}$$

*) phased array

$$E_{\vartheta_{array}} = E_{\vartheta_1} \cdot \sum_{i=0}^{n-1} e^{j i \delta} \qquad \delta = \psi - \beta_0 |\Delta\vec{r}|$$

$$= E_{\vartheta_1} \frac{e^{jn\delta} - 1}{e^{j\delta} - 1}$$

$$C_{gr} = \left| \frac{\sin\left(\frac{n\delta}{2}\right)}{n \sin\left(\frac{\delta}{2}\right)} \right| \qquad (\text{max. for } \delta \rightarrow 0)$$

Electronic noise

Definitions & properties

$$SNR = \frac{P_S}{P_N} \mapsto M_L = 10 \lg SNR$$

$$p^*(u_0) = \frac{1}{\sqrt{2\pi \tilde{u}^2}} e^{-\frac{1}{2}\left(\frac{u_0}{\tilde{u}}\right)^2}$$ … gaussin probability density function

$$P^*(u_1) = \int_{u_0=0}^{u_1} p^*(u_0) du_0 = \phi^*\left(\frac{u_1}{\tilde{u}}\right)$$ … gaussin error function

$$\phi^*(-x) = \frac{1}{2} - \phi^*(x)$$

$$\phi^*(x) \underset{|u_1| > 2\sqrt{\tilde{u}^2}}{\approx} \frac{1}{2} - \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi} \cdot x}$$

Wiener-Khichin:

$$g(\tau) = \overline{u(t) \cdot u(t+\tau)}$$ … autocorrelation

$$W_U(f) = 2 \int_{\mathbb{R}} g(\tau) \cos(2\pi f \tau) d\tau$$ … power density function

$$g(\tau) = \int_0^\infty W_U(f) \cos(2\pi f \tau) df$$

$$W_{U12}(f) = 2 \int_{\mathbb{R}} g_{12}(\tau) e^{-2\pi f \tau} d\tau$$

$$g_{12}(\tau) = \int_0^\infty W_{U12}(f) e^{j2\pi f \tau} df$$ … cross-correlation

$$\gamma = \frac{W_{U12}(f)}{\sqrt{W_{U1}}\sqrt{W_{U2}}}$$ … correlation coeff.

$$\boxed{Z_{cor} = \gamma \cdot \frac{|U_N|}{|I_N|}}$$

$$|\tilde{U}_{Nc}| = |\gamma| \cdot |U_N|$$

$$|\tilde{U}_{Nu}| = \sqrt{1 - |\gamma|^2} |\tilde{U}_N|$$

Noise uncorrelated ($\gamma = 0$): $|\tilde{U}_N|^2 = |\tilde{U}_{N1}|^2 + |\tilde{U}_{N2}|^2$

Noise fully correlated ($|\gamma| = 1$): $\tilde{U}_N = \tilde{U}_{N1} + \tilde{U}_{N2}$

Partially correlated:

$$|\tilde{U}_1| = \sqrt{|\tilde{U}_{1U}|^2 + |U_{1C}|^2}$$

$$|\tilde{U}_N| = \sqrt{|\tilde{U}_{1C} + \tilde{U}_2|^2 + |U_{1U}|^2}$$

Noise of 2-pole

noise of resistor

$$|\tilde{U}_N|^2 = W_{tU} \cdot \Delta f$$

$$\rightarrow \boxed{|U_N| = \sqrt{4kTR\Delta f}}$$

$$\overset{[\mu V]}{=} 4 \cdot \sqrt{R[k\Omega]} \sqrt{\Delta f [MHz]}$$

$$W_{tU} = \frac{h \cdot f}{k \cdot T} \frac{4kTR}{e^{hf/kT} - 1} \approx 4kTR$$

$$W_{ti} = \frac{1}{R^2} \cdot W_{tU}$$

available noise power

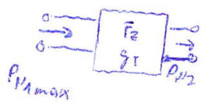
$$P_{N max} = \left(\frac{|\tilde{U}_N|}{2}\right)^2 \frac{1}{R} = kT\Delta f$$

Shot noise

$$W_{is} = 2 q_e \cdot I_{gl}$$

$$|I_S|^2 = 2 q_e I_{gl} \cdot \Delta f$$

Noise measurement



$$P_{N2} = (P_{N1max} + kT_Z \Delta f) g_T = kT_0 (m + F_Z) \Delta f g_T$$

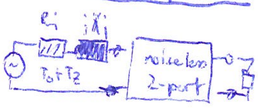
Formeln HF IV

4

A

Electronic noise

Noise of 2-ports

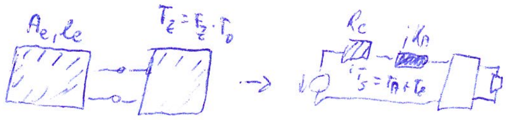


*) power match ($Z_1 = Z_i^*$) → $g_T = |A_B|^2 = \frac{P_{S_2}}{P_{S_1 max}}$ ... transducer power gain

*) $P_{N_1 max} = k T_0 \Delta f$ ... available noise power

$$P'_{N_1 max} = k T_0 \underbrace{(1 + \underbrace{\frac{T_z}{T_0}}_{F_z})}_{F} \Delta f$$

*) system noise temperature



$$P_{S_1 max} = \frac{1}{2} \frac{|E|^2 l_e^2}{4 R_a}$$

$$P'_{N_1 max} = k T_S \Delta f$$

$$R_n = \frac{|\tilde{U}_{N_V}|^2}{4 k T_0 \Delta f} \qquad G_n = \frac{|\tilde{I}_N|^2}{4 k T_0 \Delta f}$$

$\underline{Z}_{opt}$

$$\boxed{\begin{aligned} R_{opt} &= \sqrt{\frac{R_n}{G_n} + R_{cor}^2} \\ X_{opt} &= -X_{cor} \end{aligned}}$$

$$Z_{corr} = \gamma \cdot \frac{|\tilde{U}_N|^2}{|\tilde{I}_N|^2}$$

$$F_{Z_1 min} = 2 \cdot \left[G_n R_{corr} + \sqrt{G_n R_n + (G_n R_{corr})^2} \right]$$

$$\boxed{\text{*) noise matching}}$$

$$Z_{opt} = Z_L \cdot \frac{1 + \Gamma_{opt}}{1 - \Gamma_{opt}}$$

circles with const. noise figure: $\rho = \frac{Z_i - Z_{opt}}{Z_i + Z_{opt}}$ $\quad \Delta F = G_n \cdot R_{opt}$

$$F_{Z_{min}} = 1 - F_{min}$$

$$F_Z = F_{Z_{min}} + \Delta F \frac{4|\rho|^2}{1 - |\rho|^2}$$

$$|\rho| = \sqrt{\frac{F_Z - F_{min}}{F_Z + 4 \Delta F - F_{Z_{min}}}}$$

*) cascaded noise 2-ports: $n = 2$: $P_{N_3} = k T_0 \Delta f (1 + F_Z) g_{T_1} g_{T_2}$

$$F_Z = F_{Z_1} + \frac{F_{Z_2}}{g_{T_1 max}}$$

n: $$F_Z = F_{Z_1} + \frac{F_{Z_2}}{g_{T_1 max}} + \ldots + \frac{F_{Z_n}}{\prod_{i=1}^{n-1} g_{T_i max}}$$

2-port amplifiers

$$g_T = \frac{P_2}{P_{1 max}} = \frac{|S_{21}|^2 (1 - |\Gamma_i|^2)(1 - |\Gamma_L|^2)}{|(1 - S_{11}\Gamma_i)(1 - S_{22}\Gamma_L) - S_{12} S_{21} \Gamma_i \Gamma_L|^2}$$

stability circles load side $\sigma_L = \frac{(S_{22} - \det \underline{S}\, S_{11}^*)^*}{|S_{22}|^2 - |\det S|^2}$ $\qquad$ gen. side $\sigma_i = \frac{(S_{11} - \det \underline{S}\, S_{22}^*)^*}{|S_{11}|^2 - |\det \underline{S}|^2}$

$$\tau_L = \left| \frac{S_{12} S_{21}}{|S_{22}|^2 - |\det S|^2} \right| \qquad \tau_i = \left| \frac{S_{12} S_{21}}{|S_{11}|^2 - |\det S|^2} \right|$$

stable if $\dfrac{1-|S_{11}|^2-|S_{22}|^2+|\det S|^2}{2|S_{12}||S_{21}|} > 1$

& $|\det \underline{S}| < 1$

<u>Unilateral design</u>

$(S_{12} \equiv 0)$

$$g_{TU} = \frac{P_2}{P_{max}} = \frac{1-|r_i|^2}{|1-S_{11}r_i|^2} |S_{21}|^2 \frac{1-|r_e|^2}{|1-S_{22}r_e|^2}$$

$$g_{TU_{max}} = \frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$

$p_e = 1$

$\Rightarrow |S_{21}| = \sqrt{(1-|S_{11}|^2)(1-|S_{22}|^2 \frac{g}{p_i}}$

$$g_{TU} = \underbrace{\frac{(1-|r_i|^2)(1-|S_{11}|^2)}{|1-S_{11}r_i|^2}}_{p_i} g_{TU_{max}} \underbrace{\frac{(1-|r_e|^2)(1-|S_{22}|^2)}{|1-S_{22}r_e|^2}}_{p_e}$$

$\boxed{p_e = 1 \text{ due to power matching}}$

$$p = 1-|\rho|^2 = \frac{P}{P_{max}} \quad \rightarrow \rho_i = \frac{Z_i - Z_{S_{11}}^*}{Z_i + Z_{S_{11}}}$$

$$|\rho| = \sqrt{1-p} \qquad \rightarrow \rho_e = \frac{Z_e - Z_{S_{22}}^*}{Z_e + Z_{S_{22}}}$$

$$\frac{|\sigma_i|}{|\tau_i|} = \frac{|S_{11}|(1-|\rho_i|)^2)}{|\rho_i|(1-|S_{11}|^2)}$$

$\downarrow$

ρ_i berechnen

$\downarrow$

$p = 1-|\rho_i|^2$

Circles of const. mismatch: $\quad \sigma_i = \dfrac{|S_{11}|(1-|\rho_i|^2)}{1-|S_{11}\rho_i|^2}$

(analog: σ_e, τ_e with ρ_e, S_{22}) $\quad \tau_i = \dfrac{|\rho_i|(1-|S_{11}|^2)}{1-|S_{11}\rho_i|^2}$

Error estimation: $\quad \dfrac{1}{(1+\delta)^2} < \dfrac{g_T}{g_{TU}} < \dfrac{1}{(1-\delta)^2}$

$$\delta = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$